

# Optimization of Steel Structures Using Particle Swarm Technique

O. KAMAL<sup>\*</sup>, O. EL-MAHDY<sup>\*</sup>, M. Nour<sup>\*\*</sup> and G. EL-KOMY<sup>\*\*\*</sup>

<sup>\*</sup> Professor, <sup>\*\*</sup> Assistant Professor, <sup>\*\*\*</sup> Graduate Student  
Faculty of Engineering at Shoubra, Benha University, Cairo, Egypt

## ABSTRACT

The computational drawbacks of existing numerical methods have forced researchers to rely on heuristic algorithms. Heuristic methods are powerful in obtaining the solution of optimization problems. The Particle Swarm Optimization (PSO) method is a numerical optimization technique that simulates the social behavior of birds, fishes and bugs. Similar to birds seek to find food, the optimum design process seeks to find the optimum solution. In the present study, a combination between nominal PSO and Gaussian PSO is applied to get the optimum design for steel structures. A modified fly back technique is used to deal with constraints. Standard design problems selected from literatures are used to evaluate the performance of the proposed technique. The comparison showed that design obtained using the present algorithm is more efficient and economical than that provided by other design approaches.

## KEYWORDS

Particle Swarm Optimization (PSO); Steel; Frames; Design Codes; Gauss Distribution; Fly Back Mechanism

## 1. INTRODUCTION

Structural design optimization is a critical and challenging activity that has received considerable attention in the last two decades. High number of design variables, largeness of the search space and controlling great number of design constraints are major preventive factors in performing optimum design in a reasonable computation time. Despite these facts, designers and owners have always desired to have optimal structures.

The developments in the stochastic search techniques in numerical optimization have provided efficient optimum design tools for structural designers. The basic idea behind these stochastic search techniques is to simulate the natural phenomena. These stochastic search techniques do not require the derivatives of the objective function and constraints and they use probabilistic transition rules not deterministic rules [1]. The particle swarm optimization (PSO), that was only a few years ago a curiosity, has now attracted the interest of researchers around the globe.

In the present study, a combination between nominal PSO and Gaussian PSO is applied in a new technique to get the optimum design for steel structures. The development of the paper is as follows: Section 2 presents the general formulation of the particle swarm optimization approach. Section 3 literature review of PSO and structural steel design are presented. Section 4 presents the use of Gauss Distribution in PSO. In Section 5, the problem formulation and constraint handling technique are discussed. In section 7, three different case studies are analyzed to demonstrate the effectiveness of the approach in finding optimal structural optimization solutions. Finally, the paper closes with main concluding remarks.

## 2. PARTICLE SWARM OPTIMIZATION

In the simplest version of PSO, each member of the particle swarm is moved through a problem space by two elastic forces. One attracting it with random magnitude to the best location so far encountered by the particle, it is called  $L_{\text{best}}$  (local best). The other attracting it with random magnitude to the best location encountered by any member of the swarm which is typically denoted by  $G_{\text{best}}$  (global best). The position and velocity of each particle are updated at each time step until the swarm as a whole converges to an optimum [2 and 3].

The original PSO formulae as described in [4] are:

$$V_i^{t+1} = V_i^t + c_1 r_1^t (p_i^t - x_i^t) + c_2 r_2^t (p_g - x_i^t) \quad (2.1)$$

$$x_i^{t+1} = x_i^t + V_i^{t+1} \quad (2.2)$$

Where:

- $i = (1, 2, 3, \dots, N)$ , and  $N$  is the number of particles.
- $c_1$  and  $c_2$  are positive constants, referred to as cognitive and social parameters, respectively.
- $r_1^t$  and  $r_2^t$  are random vectors with components uniformly distributed in  $(0, 1)$ .
- $x_i^t = (x_1^t, x_2^t, \dots, x_D^t)$ , the location of the  $i^{\text{th}}$  particle at  $t^{\text{th}}$  iteration, and  $D$  is the number of design variables.
- $V_i^t = (V_1^t, V_2^t, \dots, V_D^t)$ , the velocity vector of the  $i^{\text{th}}$  particle at  $t^{\text{th}}$  iteration.
- $P_i^t = (P_1^t, P_2^t, \dots, P_D^t)$ , represents the best previous position vector of the  $i^{\text{th}}$  particle at  $t^{\text{th}}$  iteration.
- $P_g = (P_1, P_2, \dots, P_D)$ , the symbol  $g$  represents the index of the best particle among all the particles in the population.

The initial values of  $x_i^t$  and  $V_i^t$  can be assigned randomly. The initial value for  $P_i^t$  will be  $x_i^t$ . The initial value of  $P_g$  can be determined by evaluating all particles in the group and selecting the initial position of the best particle. Figure (1) represents the velocity updating in PSO.

Another parameter called constriction factor ( $k$ ) is introduced to insure a convergence of PSO. A simplified method of incorporating it appears in the following equations:

$$V_i^{t+1} = k[V_i^t + c_1 r_1^t (p_i^t - x_i^t) + c_2 r_2^t (p_g^t - x_i^t)] \quad (2.3)$$

$$k = \frac{2}{2 - \phi - \sqrt{\phi^2 - 4\phi}} \quad (2.4)$$

Where

$$\phi = c_1 + c_2, \text{ and } \phi > 4 \quad (2.5)$$

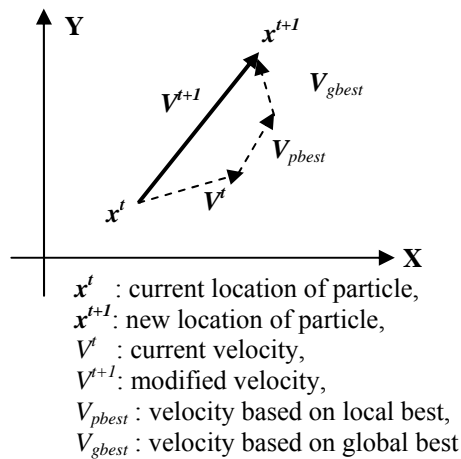


Fig. (1): PSO Position Updating

Mathematical back ground for Eqs. (2.3, 2.4 and 2.5) is widely discussed and proofed by Ozcan, Mohan, Clerc and Kennedy [5 and 6].

### 3. LITERATURE REVIEW OF PSO AND STRUCTURAL STEEL DESIGN

In the following paragraphs the most important researches who employ the PSO in the optimization of structural design are presented.

Fourie and Groenwold [7] applied the particle swarm optimization algorithm (PSOA) to the optimal design of structures with sizing and shape variables. Some new operators, namely elite particle and elite velocity, had been used.

The application of the PSO to the optimal sizing design of truss structures was studied by Schutte, and Groenwold, [8]. A simple methodology was presented to accommodate the stress and displacement constraints during initial iterations. The construction factor and dynamic inertia weight were applied to PSO and a penalty function was used as a constraints handling technique.

Bochenek and Forys [9] considered structural optimization against instability using PSO. The standard maximization of critical load was performed both for single and double buckling load. The modified optimization for post-buckling behavior was also performed.

Design and optimization of steel structures for fire resistance was also considered by Jármai, et al. [10] using modified PSO. A pressure vessel supporting frame was considered using square hollow section columns and square or rectangular hollow section for beams. Overall and local buckling constraints were considered and the steel frame was designed according to Eurocodes (1) and (3).

The early work of use of PSO to optimize the design of steel structure was presented by Heinisuo, M. et al. [11]. In their work, the design of welded steel beams for a typical structure was considered.

The tubular truss optimization using heuristic algorithm had been considered by Jalkanen [12]. The multi-criteria topology, shape and sizing optimization problem were formulated based on both material and manufacturing cost. Design constraints satisfied the requirements of steel design rules of Eurocode (3).

Kaveh and Talatahari presented a Discrete Particle Swarm Ant Colony Optimization (DPSACO) for design of steel truss [13] and steel frame [14]. The particle swarm ant colony optimization applied the particle swarm optimizer with passive congregation for global optimization and ant colony optimization worked as a local search.

Later, Kaveh and Talatahari [15 and 16] presented a hybrid algorithm based on the Particle Swarm Optimization with Passive Congregation (PSOPC), the Ant Colony Algorithm (ACO), and the Harmony Search (HS) approach to solve engineering optimization problems. The idea of ACO worked as a local search and HS utilized to handle the boundary constraints.

A refined version of particle swarm optimization technique for the optimum design of steel trusses was proposed by Doğan et al. [17]. In their work, an additional velocity term was defined and added to let the particle move randomly in certain directions in the close neighborhood of its current position, to avoid the stuck in local optimum. The penalty function technique was used to handle the ASD-AISC code constraints.

The composite steel box girder was investigated under size, shape and topology optimization using Particle Swarm Optimization by Ghasemi and Dizangian [18]. The objective function was the minimization of total weight of the structure under strength and serviceability constraints, enforced by penalty functions. All design requirements of AASHTO and Iranian Codes of Practice (ICP) for loading of bridges were considered.

A parallel version for PSO optimizer was studied by Yang and Zhang, [19] to solve large truss topological optimization problems. It was the first modified PSO that can solve truss topology optimization problem with more than 200 design variables. This modified PSO was based on  $L_{best}$  model of PSO. In their model, the information linked between different particles were set by some topology technique, i.e. one particle could share the information only with some particles like its neighbors, not with all the other ones. The  $L_{best}$  model tried to prevent premature convergence by maintaining diversity of potential problem solutions.

Particle Swarm method based optimum design algorithm for unbraced steel frames was presented by Doğan and Saka [1]. In the optimum design algorithm the design constraints were imposed in accordance with LRFD-AISC. In the design of beam-column members the combined strength constraints were considered by taking into account the lateral tensional buckling of the member. The algorithm developed selected optimum W sections for beams and columns of unbraced frame from 272 W-sections list. This selection was carried out such that design constraints imposed by the LRFD-AISC were satisfied and the minimum frame weight was obtained.

Kaveh et al. [20] introduced a new hybrid advanced algorithm by using the abilities of heuristic Particle Swarm Ant Colony Optimization (HPSACO) and a Hybrid Big Bang–Big Crunch algorithm (HBB–BC). In their study, the advantages of the HPSACO and HBB–BC were combined to improve the performance of the resulted algorithm. They considered three main steps as global searching step, local searching step and location controlling step. These

steps all together improved the exploration and exploitation abilities of the algorithm. Their proposed method was tested on frame structures from the literatures.

A modified algorithm called Accelerated Particle Swarm Optimization (APSO) was developed by Talatahari et al. [21] for finding optimum design of steel frame structures. APSO showed some extra advantages in convergence for global search. The modifications on standard PSO effectively accelerated the convergence rate of the algorithm and improved the performance of the algorithm in finding better optimum solutions.

#### 4. GAUSS-PSO COMBINATION

As a member of stochastic search algorithms, PSO has drawbacks. Although PSO converges to an optimum much faster than other evolutionary algorithms, it usually cannot improve the quality of the solutions as the number of iterations is increased [22]. Many adjustments have been made to the original algorithm that introduced in [4] over the past decade in order to improve general performance by incorporate either the capabilities of other evolutionary computation techniques. Some researchers have studied a version that eliminates the current position term, and simply places the particle at a new position based on previous successes.

One of the important modifications on PSO is the using of Gauss Distribution instead of the velocity and position update equations of PSO. It is greatly simplifies the particles swarm by stripping away the velocity rule, but performance seems not good as canonical one in some tested problems [23].

In Bare-Bones formulations of PSO, which described by Kennedy [24] particles proposed to move according to a probability distribution rather than through the addition of velocity, which called “velocity-free” PSO.

In a first study of bare-bones PSO, the particle update rule was replaced with a Gaussian distribution of mean  $(P_i+P_l)/2$  and standard deviation  $|P_i-P_l|$ , thus Eqs. (2.1) and (2.2) will be replaced by Eq. (2.6) [13 to 18]:

$$x_i^{t+1} = G\left(\frac{(P_i + P_l)}{2}, |P_i - P_l|\right) \quad (2.6)$$

where  $x_i^{t+1}$  is the position of the particle to be updated,  $G$  (mean, s.d.) is a Gaussian random number generator,  $p_i$  and  $p_l$  are the best position reached by the  $i^{\text{th}}$  particle and the best position reached by any particle in the neighborhood of the  $i^{\text{th}}$  particle, respectively.

In the present paper, the authors introduce a new strategy to deal with the Gauss Distribution and PSO by combining them in the search. First, fast search is started using the normal PSO, which give a fast trapped to local optimum after a number of iterations, then the Gaussian-PSO starts the fine tuning search. This strategy gives good results and fast convergence compared to using normal PSO only. Furthermore, Gaussian-PSO is more simple than the other hybridization methods.

## 5. PROBLEM FORMULATION AND CONSTRAINTS HANDLING

The main concept in the design of steel frames is to find the sections for columns and beams. The design should be carried out in such a way that the frame satisfies the serviceability and strength requirements specified by the design code of practice while the economy is observed in the overall or material cost of the frame. These are the design variables, constraints and objective function, respectively.

Similar to other stochastic optimization methods, the PSO algorithm is defined for unconstrained problems [25]. One of the most simple and effective method to deal with constraints is the fly-back mechanism, which purposed firstly by Hu et al. [26].

In the current study, a modified fly-back mechanism is used to handle the problem specific constraints which can be described as follows:

- 1- *During the initialization process:*
  - a. All particles are started with feasible solutions.
- 2- *When updating the global best memory ( $G_{best}$ ):*
  - a. If the new solution is infeasible, return back to the previous feasible solution.
  - b. Between two feasible solutions, the one having better objective function value is preferred.
- 3- *When updating the personal best memories ( $P_{best}$ ):*
  - a. Any feasible solution is preferred to any infeasible solution.
  - b. Between two feasible solutions, the one having better objective function value is preferred.
  - c. Between two infeasible solutions, the one having smaller sum of constraint violation is preferred.

This modified technique is powerful in dealing with constraints and more simplified from other techniques such as penalty function, as no additional parameters are needed for the applied constraints – handling technique.

The boundary conditions are the limitations of sections area according to slandered tables of steel sections, or the thicknesses of steel plates available in the market in case of built up section. In the present work, before updating memories and check constraints, each design variable passes through a filter which works as follows:

- 1- Pass the design variable if it satisfying the boundary conditions.
- 2- Otherwise, regenerate the design variable to satisfying the boundary conditions.

Algorithmic flow chart of the proposed technique is shown in Fig. 2

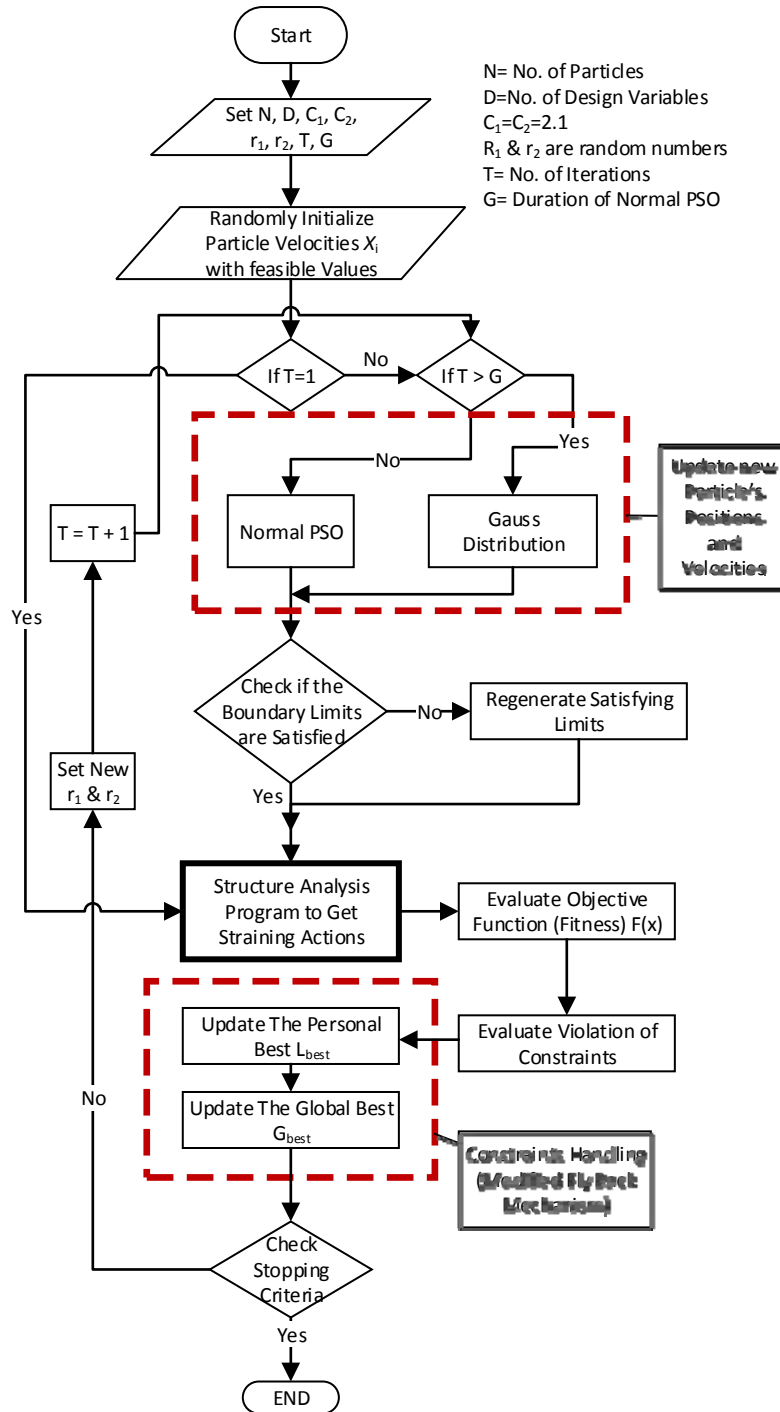


Fig. (2): Algorithmic Flow Chart

## 6. NUMERICAL EXAMPLES

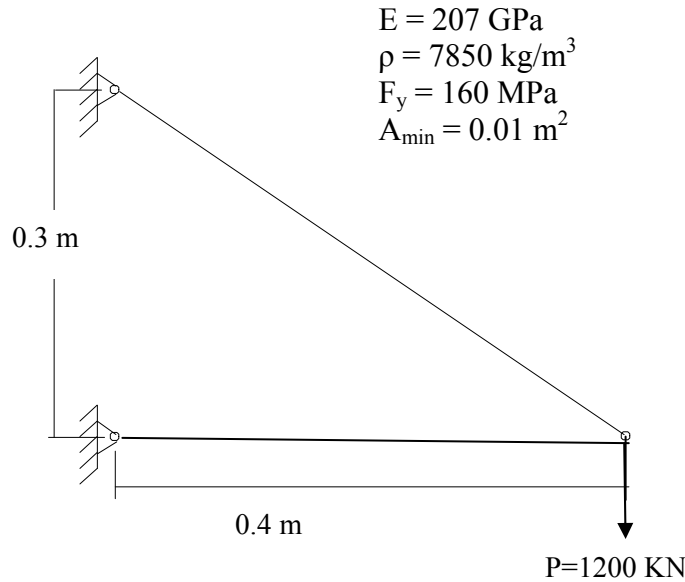
To demonstrate the efficiency of the algorithm, three well-known examples selected from the literatures are designed using the proposed method. In these designs, the duration of the normal PSO is considered as the number of iterations in which the algorithm use PSO in search before starting Gauss search (denoted as G). The value of  $C_1$  and  $C_2$  in Eq. (2.1) are considered to be 2.1 and construction factor  $K$  is used.

**Example No. (1): Two Bars Truss**

The first example is a benchmark example introduced in [7 and 27]. It is a simple truss structure depicted in Fig. 3. The member cross sections represent the design variables. Arora [27] gives an analytical solution of  $f = 8.046 \times 10^3$  kg which is the same truss weight obtained using the purposed algorithm. A number of trials has been done to investigate the effect of the duration of normal PSO, each trial has 400 iterations and take less than 5 minutes. The convergence history is showed in Table 1 and Figs. 4 and 5.

**Table 1: Iterations Results for Example No. (1)**

| Trial No. | No. of Iterations (T) | No. of Particles (N) | Duration of PSO (G) | Start Weight (Kg) | Optimum Weight (Kg) | Iteration of Optimum Weight |
|-----------|-----------------------|----------------------|---------------------|-------------------|---------------------|-----------------------------|
| 1         | 400                   | 5                    | 50                  | 14822.19          | 8046.24             | 150                         |
| 2         | 400                   | 5                    | 100                 | 18061.35          | 8046.31             | 234                         |
| 3         | 400                   | 5                    | 150                 | 16109.72          | 8046.24             | 291                         |
| 4         | 400                   | 5                    | 200                 | 15771.68          | 8046.38             | 259                         |
| 5         | 400                   | 5                    | 100                 | 14822.19          | 8046.28             | 225                         |
| 6         | 400                   | 10                   | 100                 | 14577.46          | 8046.24             | 145                         |
| 7         | 400                   | 15                   | 100                 | 10699.74          | 8046.24             | 152                         |
| 8         | 400                   | 20                   | 100                 | 11586.28          | 8046.24             | 128                         |



**Fig. (3): Two Bars Truss**



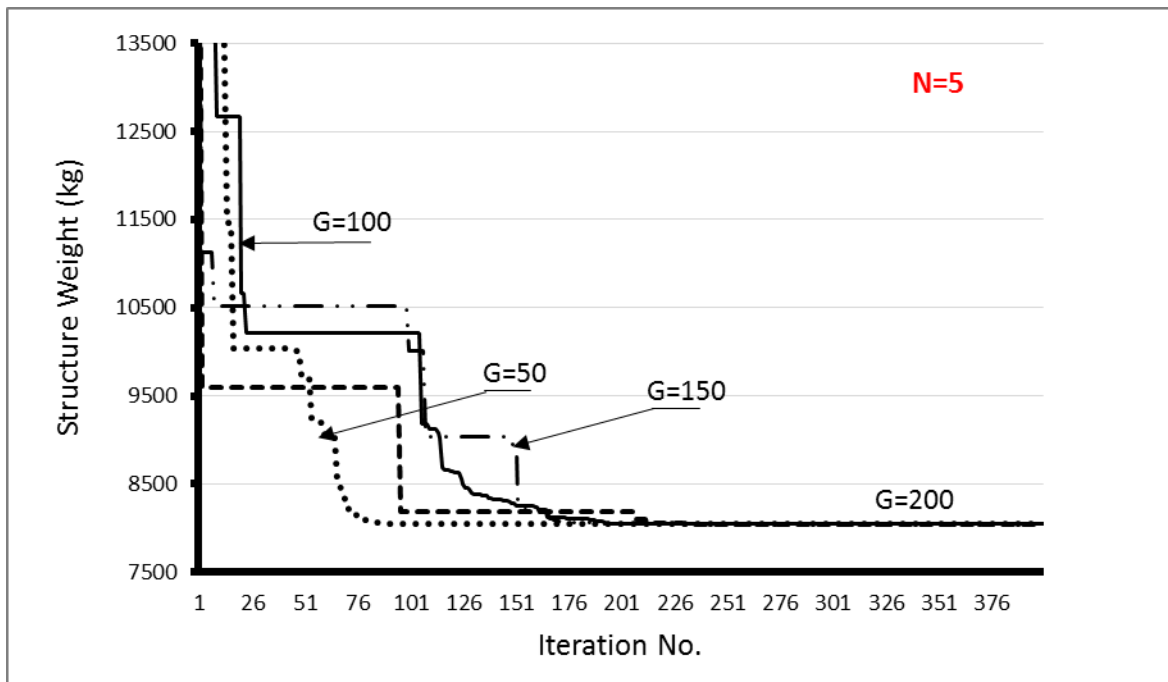


Fig. (4): Iteration History for Example No. 1.  
(No. of particles =5 and changing the duration of PSO search)

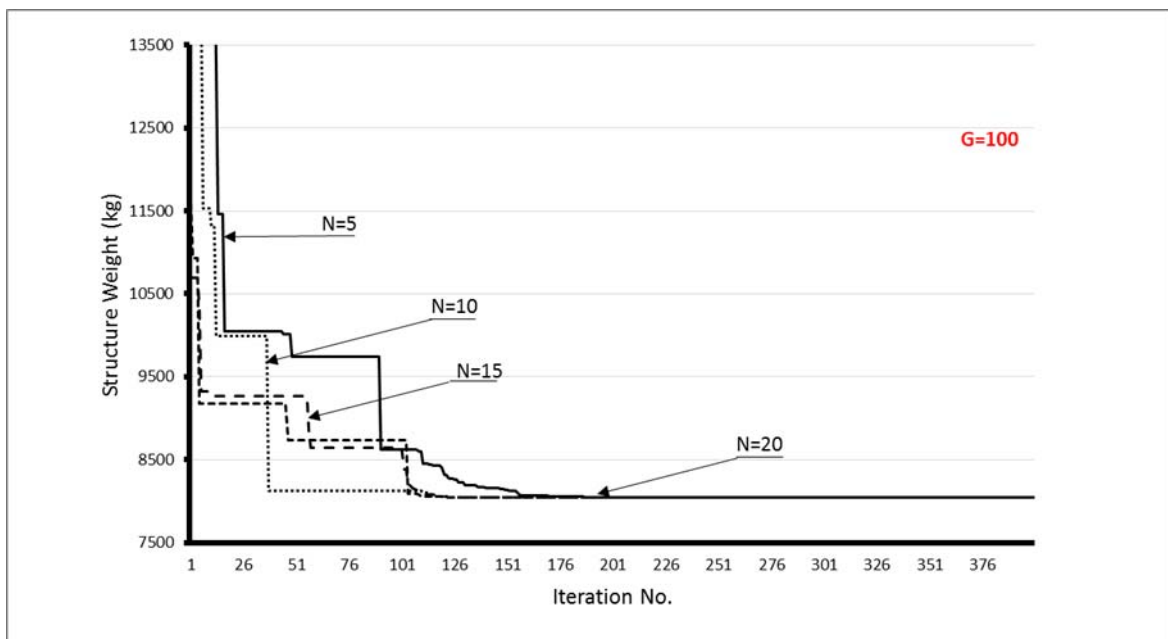


Fig. (5): Iteration History for Example No. 1.  
(No. of particles varied and the duration of PSO search is 100 iterations)

**Example No. (2): Two – bay Three Storey Steel Frame**

This Frame was considered by Kaveh and Malakoutirad [28]. Figure (6) shows the configuration and applied loads of the frame. The 15 members of the structure have been categorized into seven groups. Each group has the same cross section and represents a design variable as indicated in the figure.

The material properties are:

$$E = 200 \text{ GPa}$$

$$\rho = 7850 \text{ kg/m}^3$$

$$F_y = 235 \text{ MPa}$$

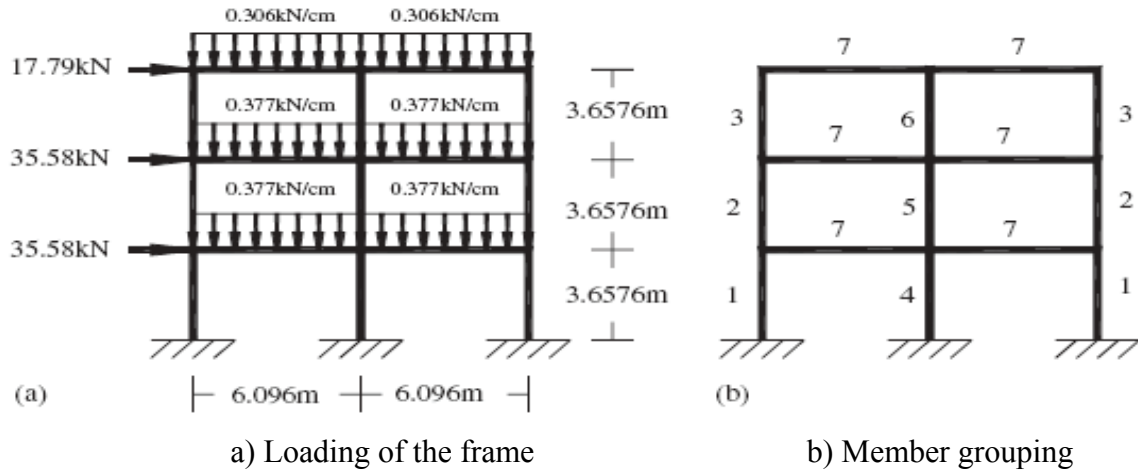


Fig. (6): Two-bay Three-Storey Frame

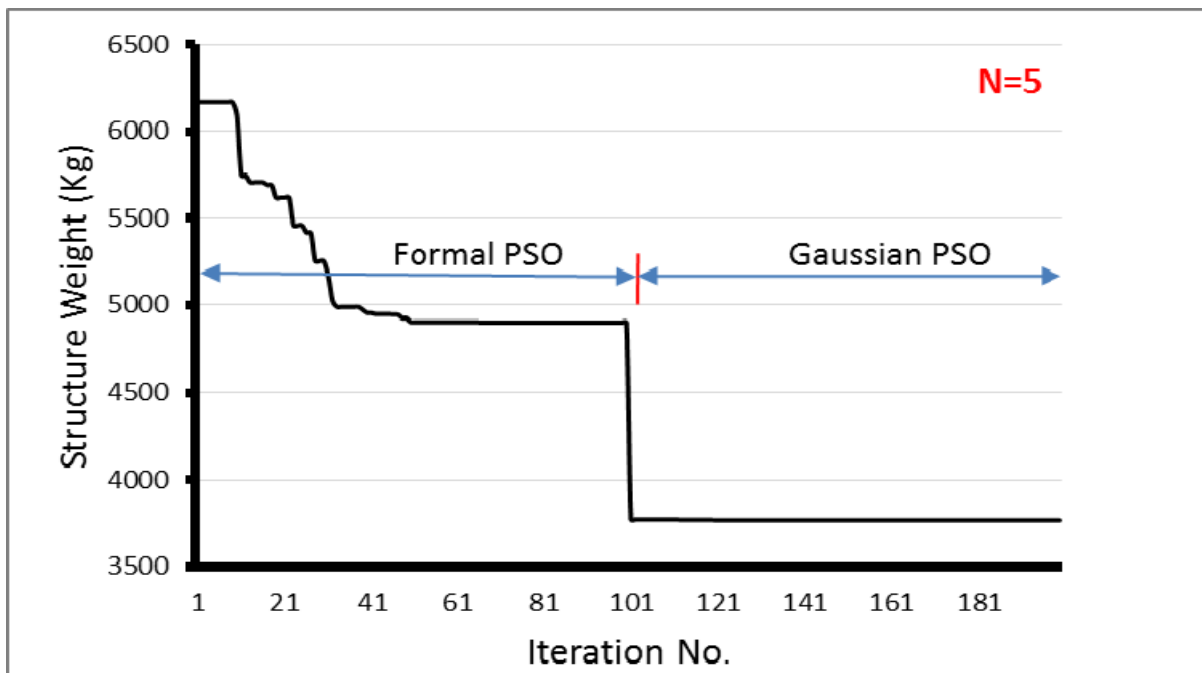


Fig. (7): Iteration History for Example No. 2.  
(No. of particles = 5 and the duration of PSO search is 100 iterations)

The constraints include the design specifications of AISC-ASD for each member. The allowable inter-storey drift is 12 mm and allowable sway of the top storey is 36 mm. The population sizes considered by Kaveh was 425 particles and 5000 iterations were used. Figure (8) shows the iteration history of the compared example.

Figure (7) shows the iteration history for this example using current purposed algorithm. This example demonstrates the effeteness of Gauss distribution on the search. In this work, only 5 particles and 200 iterations are used to obtain the same results with a total running time less than 26 minutes.

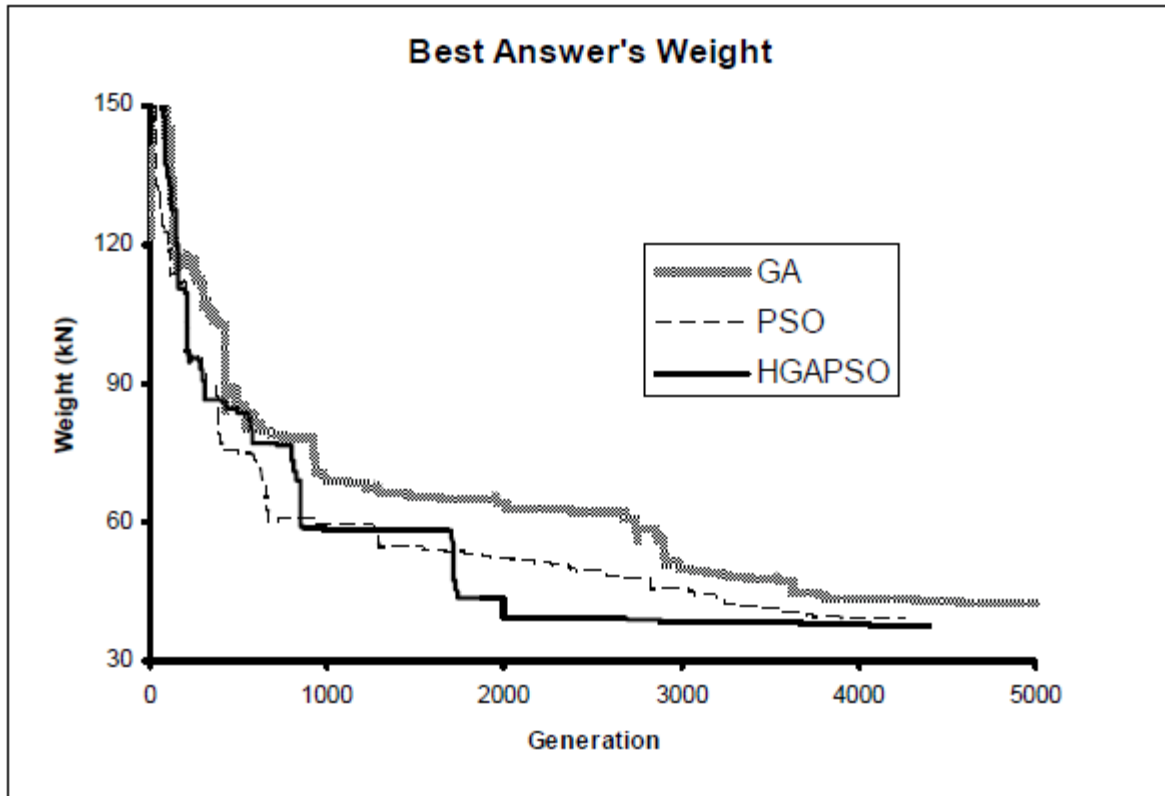


Fig. (8). Iteration History for Example No. 2 as shown in [28]

### ***Example No. (3): Two – bay Six Storey Steel Frame***

The third example is two-bay, six storey steel frame shown in Fig. (9). This frame was considered firstly by Kameshki [29] using GA, and Kaveh et al. [16] using Big – Bang algorithm and later considered by Dogan and Saka [1] using PSO. The frame consists of thirty members that are collected in eight groups as shown in the figure. The allowable inter-storey drift is 12 mm while the lateral displacement of the top storey is limited to 72 mm. The modulus of elasticity is 200 GPa.

According to Dogan and Saka, the optimum design is determined after 6500 iterations and the minimum weight of the frame is 7533 kg. In the present work, an optimum weight of 7047 kg is obtained after only 250 iterations using 15 particles. Figure (10) shows the iteration history of the design.

## **7. CONCLUSIONS**

A particle swarm optimizer and Gaussian-PSO are combined to develop an optimum design algorithm for steel frames. Constraints are handled with modified fly-back mechanism. The design algorithm is mathematically quite simple and effective in finding solutions of combinatorial optimization problems. The PSO is acting as fast optimizer and the

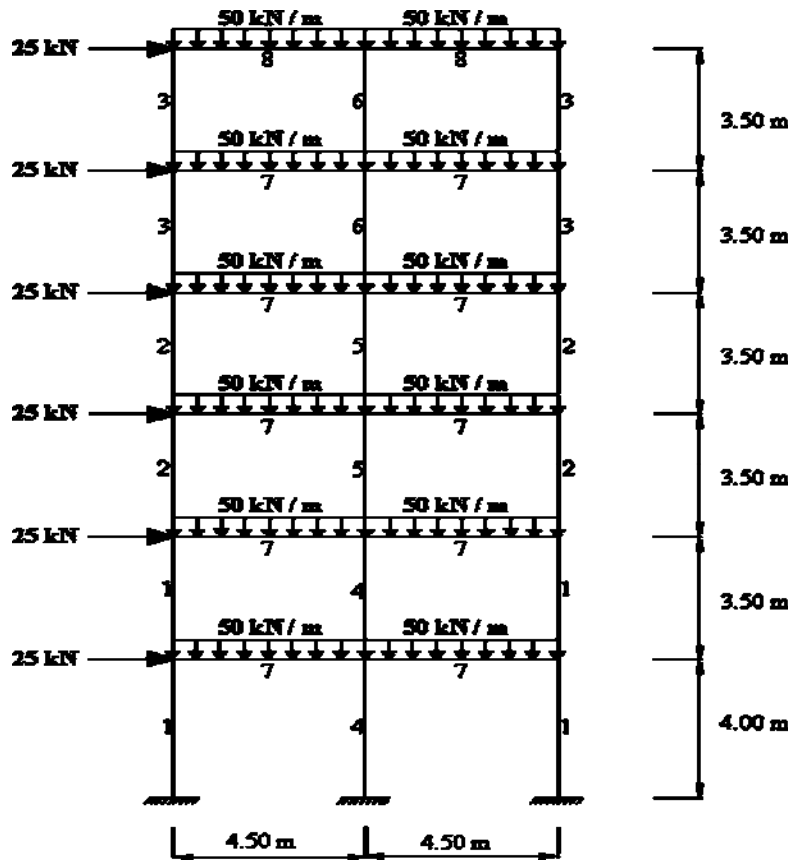


Fig. (9). Two-bay Six-Storey Frame

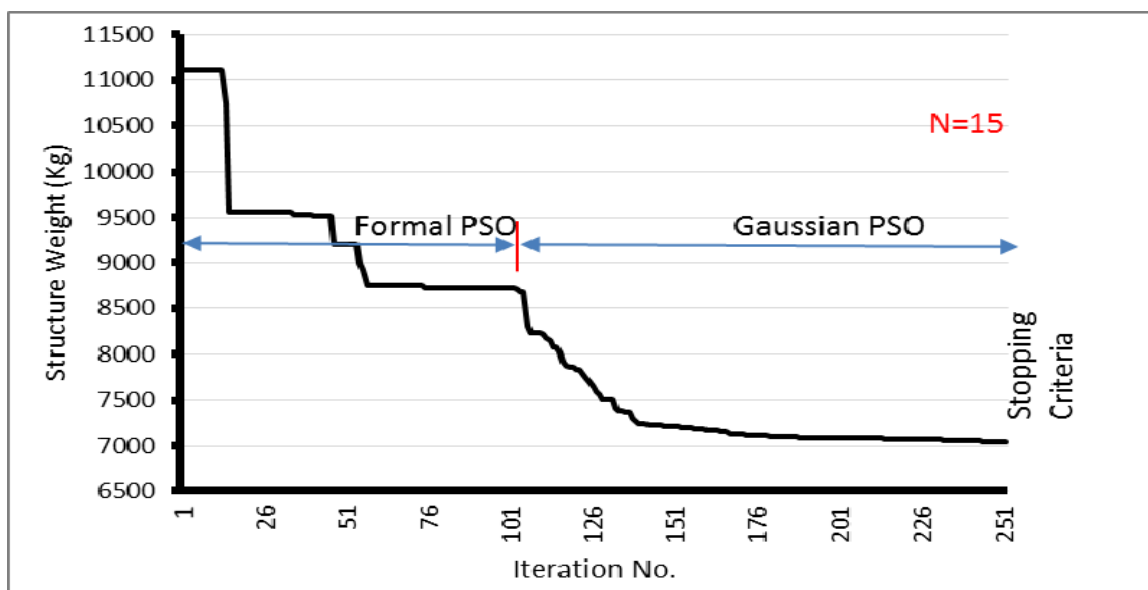


Fig. (10): Iteration History for Example No. 3.  
(No. of particles = 15 and the duration of PSO search is 100)

Gaussian-PSO plays the fine tuning search. The technique reduces the number of iterations required to reach the optimum design (100 to 400 iterations) and also allow using smaller number of particles (5 to 15 particles). The studied examples show that 100 iterations are enough for PSO fast search before starting Gaussian-PSO. Moreover, the studied examples show that the modified fly-back mechanism is powerful in particle swarm optimization technique.

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